Technical Comments

Comment on "Completely Automatic Weight Minimization Method for High-Speed Digital Computers"

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IN Refs. 1 and 2 an automated method for minimum weight design for structures has been put forward and illustrated. The method employed is an implementation of the method of constrained steepest descent which disregards "cornering," a well known pitfall of this method.

The structural design problem is viewed as an inequality constrained minimization problem in an m dimensional orthogonal space such that each point in the space $(x_1, x_2, \ldots x_m)$ represents a unique design. Upper and lower limits on design variables $(x_1, x_2, \ldots x_m)$ as well as upper and lower limits on behavior variables $(\sigma_1, \sigma_2, \ldots \sigma_t)$ are provided for, and a multiplicity of fixed load conditions is included. The problem in Refs. 1 and 2 is essentially the same as the structural synthesis problem which has been studied extensively by this author and his co-workers.[3-10]

In Refs. 1 and 2 it is stated that a feature of prime importance in the method presented is that, "after a gap has been closed subsequent $\bar{u}$'s are selected so that further changes in $\bar{x}$ have no effect upon that gap." That this is not a valid approach is seen by noticing that if there are m design variables $(x_1, x_2, \ldots x_m)$ and t inequality constraints, where $t > m$ (which is usually the case in structural synthesis), there will be more than one point in the space where m constraint hypersurfaces intersect (i.e., more than one corner point) regardless of the weight function. In addition, there is no reason to expect that the first constraint encountered (gap closed), or any of the others as they are encountered in order, will be the active constraints at the optimum design point. This is illustrated in the hypothetical two dimensional design variable space shown in Fig. 1. Points $\bar{x}_1$ and $\bar{x}_2$ in Fig. 1 are corner points (i.e., points where the number of gaps closed equals the number of design variables $r = m = 2$); however it is apparent that the design represented by $\bar{x}_3$ is the optimum design. The method of Refs. 1 and 2 when started from $\bar{x}_a$ yields $\bar{x}_1$ as the proposed optimum design, when started from $\bar{x}_b$ yields $\bar{x}_2$ as the proposed optimum, and when started from $\bar{x}_c$ yields $\bar{x}_3$ as the proposed optimum design. However, it is clear that $\bar{x}_3$ is the only optimum design for the hypothetical case depicted in Fig. 1.

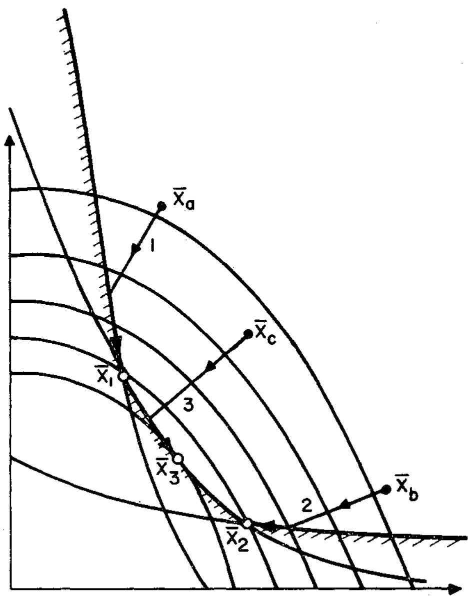

Fig. 1 Hypothetical design space.

In Ref. 11 the difference between the method of constrained steepest descent with equality constraints is clearly distinguished from the method of constrained steepest descent with inequality constraints. In particular, the step direction in Refs. 1 and 2 is found by seeking u such that

$$g^T u \to \min \tag{1}$$

with

$$u^T u = 1 \tag{2}$$

$$C^T u = 0 \tag{3}$$

whereas, in fact, the step direction should be found by seeking u such that

$$g^T u \to \min \tag{1'}$$

with

$$u^T u = 1 \tag{2'}$$

$$C^T u \geq 0 \tag{3'}$$

The inequality sign in Eq. (3′) is of cardinal importance because it permits opening previously closed gaps in the quest for lower weight designs.

In Refs. 11 and 12 it is shown that finding the direction of steepest descent subject to inequality constraints [i.e., the programing problem stated in Eqs. (1′–3′)] is equivalent to a quadratic programing problem that has a unique solution. Another approach that may be used to overcome the "cornering" difficulty is to employ an alternate step procedure[3-9] in which any acceptable design, having the same or lower weight than the current cornered point, is sought, thus avoiding the premature termination of the synthesis path.

It is interesting to note that the problem of seeking the fully stressed design of elastic redundant trusses, subject to a multiplicity of load conditions, also does not in general yield minimum weight designs for a similar reason. In Ref. 13 it is assumed that the minimum weight design has the characteristic that each member is fully stressed in at least one load condition. That seeking a fully stressed design does not necessarily lead to the minimum weight optimum design for stressed, limited, statically indeterminate trusses, subject to a multiplicity of load conditions, can be seen from the following argument. View the synthesis problem using the concept of a design space.† For trusses of fixed configuration where the cross-sectional area of each member is considered as an independent design variable, the number of design variables equals the number of members m. When each member is fully stressed in at least one condition, the design lies at the intersection of m constraint hypersurfaces (i.e., a corner). As pointed out in Ref. 3, there is no reason why such corners should necessarily be points representing designs of minimum weight.

Received August 13, 1964. *Editor's Note:* The author of the paper being discussed was invited to write a reply but declined

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† An m dimensional orthogonal space in which there is a coordinate axis for each design variable $(x_1, x_2, \ldots x_m)$. Each point in this space represents a unique design.

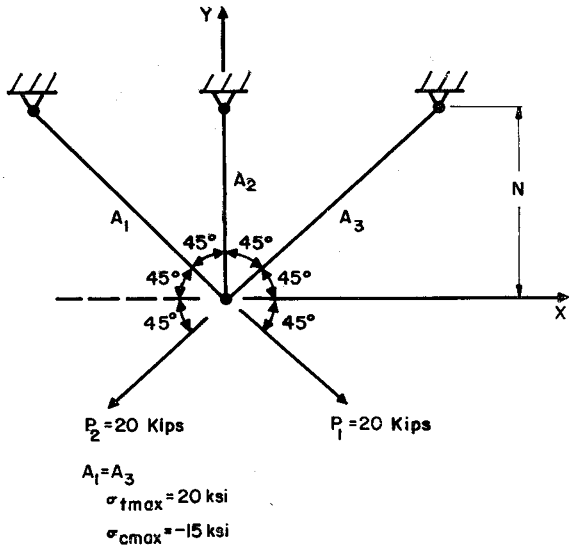

Fig. 2 Numerical example.

Consider the following simple example (see Fig. 2). Given a symmetric 45° planar truss, subject to two distinct loads conditions ($P_1 = 20$ kips and $P_2 = 20$ kips), determine the cross-sectional areas of members A_1 and A_2 for minimum weight. Let all members be made of the same material having a weight density ρ; assume the maximum allowable tensile stress in any member in any load condition is 20 ksi, and the maximum allowable compressive stress is -15 ksi. Assume for this example that displacement limits are not critical and may be ignored. Let σ_{ij} denote the stress in the ith member in the jth load condition. The equations of the constraint surfaces are easily found:

$$\sigma_{11}(A_1,A_2) = \sigma_{32}(A_1, A_2) = 20 \qquad (4)$$

is equivalent to

$$A_2 = (2)^{1/2}A_1(1 - A_1)/(2A_1 - 1) \qquad (4')$$

$$\sigma_{21}(A_1, A_2) = \sigma_{22}(A_1, A_2) = 20 \qquad (5)$$

is equivalent to

$$A_2 = (1 - A_1)/2^{1/2} \qquad (5')$$

and

$$\sigma_{12}(A_1, A_2) = \sigma_{31}(A_1, A_2) = -15 \qquad (6)$$

is equivalent to

$$A_2 = 3(2)^{1/2}A_1^2/(4 - 6A_1) \qquad (6')$$

The weight function is

$$W = \rho N[2(2)^{1/2}A_1 + A_2] \qquad (7)$$

The significant portion of the design variable space for the example case is shown in Fig. 3. The minimum weight optimum design is represented by point 3 in Fig. 3 ($A_1 = 0.788$, $A_2 = 0.41$, $W/\rho N = 2.64$). However, if the method of Refs. 1 and 2 is employed, with an initial design in the region A, then the proposed optimum design found will be represented by point 1 in Fig. 3 ($A_1 = 0.572$, $A_2 = 2.40$, $W/\rho N = 4.02$), whereas, with an initial design in the region B, the proposed optimum design found will be represented by point 2 in Fig. 3 ($A_1 = 1.0$, $A_2 = 0$, $W/\rho N = 2.83$). If the initial trial design lies in the region C, then the method of Refs. 1 and 2 will find the optimum design represented by point 3 in Fig. 3. It is interesting to note that procedures aimed at finding a fully stressed design[13] will terminate a point 2 starting from any acceptable initial design. Alternate step methods will converge to the optimum design starting from any acceptable initial design. The method of Refs. 1 and 2, modified as suggested herein [see Eqs. (1′–3′)], will also converge to the optimum design starting from any acceptable initial design.

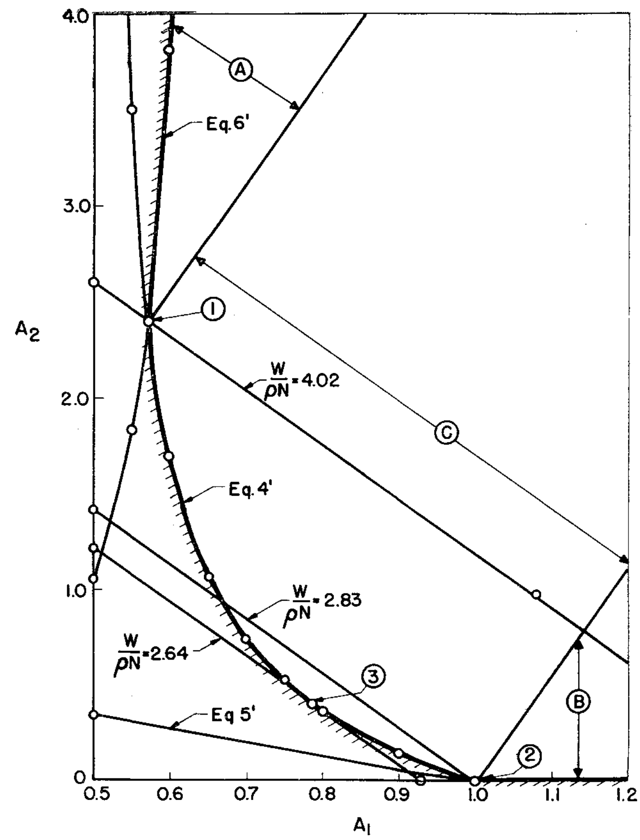

Fig. 3 Example problem design space.

In conclusion, the method presented in Refs. 1 and 2 is an implementation of the method of constrained steepest descent which disregards "cornering." Depending upon the initial trial design, it may terminate at the local optimum design, or it may yield a design that is critical in m respects, which is not necessarily the local optimum design. With the modification suggested in Eqs. (1′–3′) the method will be free to seek the local optimum design, or, in the absence of relative minima, the optimum design. The question of the relative efficiency of the modified method in comparison with other methods remains, for the present, an open question.

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Further Comment on "Low-Altitude, High-Speed Handling and Riding Qualities"

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A'HARRAH'S reply[1] to my technical comment[2] could be answered point by point with a painstaking exposition of the underlying considerations deemed important by myself and others in the understanding and analysis of pilot induced oscillation (PIO). Reference 3, containing such an exposition, has been forwarded to A'Harrah for his private consumption; he may or may not consider it pertinent to his particular tests. (It does point out, citing experimental data, that when attempting to control the sinusoidal inputs that exist in a real PIO situation, the pilot describing function time delays associated with random-appearing or discrete inputs largely disappear.) In the meantime, to put our public argument to rest independently of analyses based on assumed linearized pilot models, I shall take refuge behind some recent data published by A'Harrah's colleagues.[4]

Figure 1 presents airplane characteristics investigated in the same simulation facility used by A'Harrah. These characteristics, obtained from a consistent set of airplane derivatives, are superposed on the now familiar acceptance boundary plot[5] in Fig. 1, taken directly from Ref. 4 (Fig. A7). For the identified test points, the values of $-Z_w \doteq 1/T_{\theta_2}$ were 0.937 and 1.25 for $M = 0.9$ and 1.2, respectively; in contrast, the data used to define the original acceptance boundaries, including the PIO limit lines, were obtained for $-Z_w = 3.22$, as noted in the comment.[2] Also, the stick-force/g for both Mach numbers was 1.9; and the friction band was 4 lb, as opposed to the 1.2-lb breakout of the Ref. 5 tests. The terrain-following task was similar and the gust input intensities were, if anything, greater (rms gust velocities of up to 20 fps). In addition, one series of 90-min "flights" involved four augmenter "failures" from maximum augmentation (Δ symbol) to the basic airplane (0 symbol). I quote:

> In summary, the basic airplane was only marginally unsatisfactory, so that even though the pitch augmented tests showed improved longitudinal characteristics, as would be expected, the actual failure of the augmenter caused no serious control problem. The pilot noticed that the changes from the damped to the undamped mode were felt mainly in a slight change in characteristics of the display, and also to a small degree in seat movements. *Pilot-induced oscillations were never present*[4] (my italics).

The characteristics of one of the basic airplanes lie on the "PIO limit" for 5 lb/g and the other on the limit for 1 lb/g (Fig. 1); hence, the absence of PIO's in these cases cannot be ascribed to the stick force characteristics (1.9 lb/g) that lie between these values. It seems plausible, then, to consider that perhaps the changes in $1/T_{\theta_2}$ are indeed significant. But A'Harrah argues[1] that for his PIO boundaries $1/T_{\theta_2}$ could not possibly be significant because "...for the flight conditions under discussion here, the aircraft attitude change is extremely small relative to changes in normal acceleration or rate of climb"; and later, "...the all-attitude-indicator pitch indication [is] useless to the pilot for the type of precision flying during which a PIO might be excited." However he states in Ref. 5 that "the altitude tracking (task) tended to filter the short-period dynamics per se" but that "by requesting the pilots to maneuver the aircraft as they would while making corrections in close formation flying or in any tight spot where precision control of load factor or *pitch attitude* (my italics) is critical, PIO characteristics were readily apparent...." The latter statement (contained in the paper[5]) does not rule out the possibility of pitch attitude control, whereas the former (contained in the reply[1]) does.

Received September 3, 1964.

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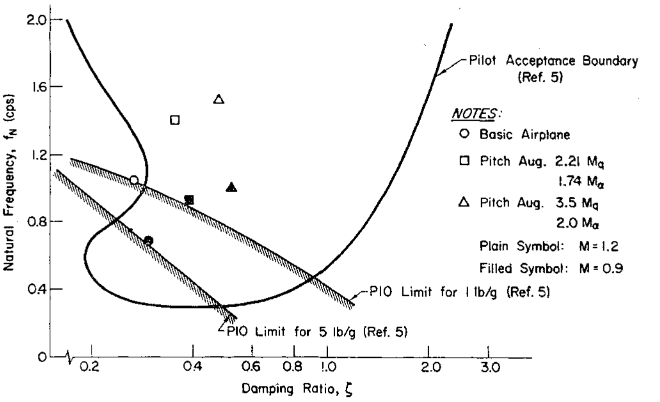

Fig. 1 Longitudinal short-period stability characteristics center stick control (Fig. A7 of Ref. 4).

For the Ref. 4 tests cited previously, the usefulness of the all-attitude-indicator (AAI) is apparently less open to question; the two pilots who commented in this connection had this to say:

Pilot 4: "Pitch indicator of AAI helped very little in pitch control. Too little sensitivity for making pitch corrections. Thought so at first, but have now learned to use it."

Pilot 8: "The aircraft responses were clearly visible on the AAI and they appeared to correspond closely to anticipated motion in 5 degrees of freedom."

Further, as noted in the Ref. 4 conclusions quoted above, the pilots *were* able to distinguish changes in short-period characteristics and the basic airframe was rated correctly (i.e., only marginally unsatisfactory, corresponding closely to its position on Fig. 1 and on its location roughly midway in the unsatisfactory lateral-directional region[4]), so the task did not apparently tend "to filter the short-period dynamics per se." This may have been due to the somewhat more sophisticated display-director commands used.

We have, then, two sets of data run in the same facility under similar environments for similar tasks. One set[4] generated by using self-consistent values of $1/T_{\theta_2}$ and $\zeta\omega$ indicated no PIO situations; the other set[5] showed a PIO region for values of $\zeta\omega$ inconsistent[2] with the *fixed* value of $1/T_{\theta_2}$ employed. In the face of these facts, the arguments advanced in Ref. 2, which go a long way toward explaining these different results, appear highly pertinent.

References

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